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Stabilized turbulent fluid friction and heat transfer in circular tubes with internal sand type roughness at moderate Prandtl numbers

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Abstract—A mixing length type model is proposed and numerical results are reported for the Fanning friction factor and the heat transfer coefficient—the Nusselt number—for hydrodynamically and thermally stabilized single phase incompressible turbulent forced convection flow in a round tube with internally sand-roughened walls for moderate Prandtl numbers. Both constant wall heat flux and wall temperature boundary conditions are simulated. A comparison between present results and other experimental and numerical data available indicates better agreement compared to calculations published earlier.

INTRODUCTION

SEVERAL models [1–8] were published for the numerical prediction of friction factors and heat transfer coefficients of flows over roughened walls. All of them are based on the concept of 'mixing length' [9] while different methods are used to account for the influence of the roughness height on the 'mixing length' and the effect of the roughness on the thermal resistance near the wall.

A 'mixing length' approach was also adopted in refs. [10, 11] where elements of several previously published models were combined to yield a model of stabilized turbulent single phase incompressible momentum and heat transfer in smooth round tubes. A novel feature of this model is the automatic calculation of the thickness of the wall region for any particular set of flow parameters. Tables of Fanning factors and Nusselt numbers are reported and compared to similar data published elsewhere to illustrate the validity of the model.

The purpose of this paper is to extend the mixing length model [10, 11] for the case of time-independent stabilized flow in round tubes with sand-grained roughness which can be used to predict in-tube Nusselt numbers, Fanning friction coefficients and mean value boundary layer profiles. Friction factors and constant wall heat flux Nusselt numbers will be compared to the experimental data [1, 2], respectively, to validate the model. Numerical values of constant wall temperature Nusselt numbers for internally sand-grained tubes as far as the authors are aware, have not yet been published; therefore, the latter results are expected to be new.

In what follows dimensionless variables will be used unless the opposite is explicitly stated. Particular scaling factors and variables used are explain in the Nomenclature.

THE PHYSICAL MODEL

Physical evidence has shown [3] that some turbulent flows can be separated into two regions of different transport properties: (1) the wall region $0 \leqslant \eta \leqslant \eta_w$ where the whole influence of the wall is exhibited and (2) the core region $\eta_w < \eta \leqslant 1$ which is not influenced by the roughness of the wall if the height of the roughness elements $k_s/D < 0.05$. In this case the thermal influence of the wall on the flow is exerted through the thickness of the viscous sublayer which is the main thermal resistance between the fluid and the wall. When the flow regime changes from hydrodynamically smooth ($k_s^+ < 5$) through a transient state ($5 \leqslant k_s^+ \leqslant 70$) to a fully rough regime ($k_s^+ > 70$) the viscous sublayer becomes thinner and for $k_s^+ > 70$ it is considered completely destroyed. In the fully rough regime a very thin film of fluid surrounding the roughnesses exists where the heat is transferred by conduction. This film is the main thermal resistance and therefore it cannot be ignored but becomes less important in the transient regime as the viscous sublayer begins to develop and enclose the roughness elements. Following ref. [6] this conduction film is modeled here as a temperature step at the wall.

From the above it follows that the mathematical model of the process has to reflect both the change of the viscous sublayer and the presence of the additional thermal resistance.

NOMENCLATURE

a_f	thermal diffusivity [$m^2 s^{-1}$]
B^+	constant, equation (5e)
D	tube diameter [m]
f	Fanning friction factor, equation (4)
k_s	roughness height [m]
k_s^+	k_s/L^*
L^*	ν/U^* [m]
L^+	mixing length, equation (2b)
Nu_f	in-tube Nusselt number, $\alpha_f D/\lambda_f$
Pr_f	Prandtl number, ν/a_f
Pr_t	turbulent Prandtl number, equations (5d) and (5e)
q	diffusive heat flux, equation (5b)
R	$D/2$ [m]
R^+	R/L^*
Re_f	in-tube Reynolds number, $u_{av}D/\nu$
r'	radial flow coordinate, $0 \leqslant r' \leqslant R$ [m]
r	r'/R
St_f	in-tube Stanton number, $Nu_f/(Pr_f Re_f)$
St_w	defined by equation (9)
U^*	friction velocity, $(\tau_w/\rho)^{0.5}$ [$m s^{-1}$]
U^+	u/U^*
u	in-tube flow velocity [$m s^{-1}$]
x	axial flow coordinate [m]
y	wall coordinate, $1-r'$ [m].

Greek symbols

α, β	constants, equation (6c)
α_f	in-tube heat transfer coefficient [$W m^{-2} K^{-1}$]
ε_h/ν	turbulent eddy diffusivity, equation (5c)
ε_m/ν	turbulent eddy viscosity, equations (2a)–(2d)
η	$1-r = y/R$
$\Delta\eta$	$\Delta y/R$
η_w	thickness of the wall region
θ	dimensionless temperature
θ_0	defined by equation (6b)
λ_f	fluid thermal diffusivity [$W m^{-1} K^{-1}$]
μ^2	eigenvalue, equation (8a)
ν	fluid kinematic viscosity [$m^2 s^{-1}$]
ξ	$4(x/D)/(Re_f Pr_f)$
ρ	fluid density [$kg m^{-3}$]
τ	shear stress [$kg m^{-1} s^{-2}$]
τ_w	wall shear stress [$kg m^{-1} s^{-2}$]
Φ	defined by equation (6c)
Ψ	eigenfunction, equations (8).

Superscripts

*	scaling factor, containing τ_w
+	scaled by L^* or U^*
H	refers to constant wall heat flux
T	refers to constant wall temperature.

Subscripts

f	fluid
h	heat
m	momentum
w	wall.

THE MATHEMATICAL MODEL AND ITS SOLUTION

The results to be reported are obtained if the following assumptions hold: (a) the fluid is single phase, incompressible and its physical properties are constant; (b) the transport processes are time independent; (c) the turbulent flow is both hydrodynamically and thermally stabilized; (d) the effects of heat generation and axial diffusion in the flow are negligible. Under these assumptions the momentum and heat transport balance equations can be decoupled and studied separately.

The momentum transport

For the particular case considered the momentum balance yields the linear shear stress distribution

$$\tau/\tau_w = 1-\eta \tag{1a}$$

where the shear stress τ/τ_w is assumed to be

$$\frac{\tau}{\tau_w} = \frac{1}{R^+}\left(1+\frac{\varepsilon_m}{\nu}\right)\frac{dU^+}{d\eta}. \tag{1b}$$

Following ref. [10], for the wall region the latter will be defined as

$$\frac{\varepsilon_m}{\nu} = R^+\left(\frac{L^+}{R^+}\right)^2\frac{dU^+}{d\eta}, \quad 0<\eta<\eta_w \tag{2a}$$

$$\frac{L^+}{R^+} = 0.4\eta(1-\exp(-\eta R^+/26)). \tag{2b}$$

In equations (2) η_w is the unknown thickness of the wall region. To insert corrections reflecting the influence of the roughness elements in equations (2), the approach of Rotta developed in detail for a rough plate [3] will be used. In this model the effect of roughness is considered to be equivalent to a velocity jump over the viscous sublayer and it can be represented by a shift of the smooth flow velocity profile. For a flow over a rough wall the reference wall is shifted downward by an amount Δy and it moves with a velocity Δu in a direction opposite to the direction of the main flow [3]. Here this quantity is represented by $\Delta\eta$ [3]

$$\Delta\eta = 0.9\frac{1}{R^+}((k_s^+)^{0.5}-k_s^+\exp(-k_s^+/6)). \tag{2c}$$

Equation (2c) has been obtained by correlating the experiments [1]. Thus the wall roughness can be accounted for by formally replacing η by $(\eta+\Delta\eta)$ in equations (1a) and (2b).

In the core region there will be no correction for the wall roughness, therefore [10, 12]

$$\frac{\varepsilon_m}{\nu} = 0.07044R^+(1-(1-\eta)^2)(1+2.345(1-\eta)^2), \quad \eta_w \leqslant \eta \leqslant 1. \quad (2d)$$

Combining equations (1) and (2), taking into account the axial symmetry of the flow and a no-slip condition for the velocity at the tube wall, one has the following initial value problem:

$$\frac{dU^+}{d\eta} = \begin{cases} \dfrac{2R^+(1-(\eta+\Delta\eta))}{1+\left(1+4(R^+)^2(1-(\eta+\Delta\eta))\left(\dfrac{L^+}{R^+}\right)^2\right)^{0.5}}, & 0<\eta\leqslant\eta_w \\ \dfrac{R^+(1-\eta)}{1+0.07044R^+(1-(1-\eta)^2)(1+2.345(1-\eta)^2)}, & \eta_w<\eta\leqslant 1 \end{cases} \quad (3a)$$

$$U^+ = 0, \quad \eta = 0 \quad (3b)$$

to determine the velocity $U^+(\eta)$, R^+ and η_w. For this purpose a continuity condition on ε_m/ν is imposed at $\eta=\eta_w$ and an iterative process is organized to converge on the value of R^+. Once equations (3) are solved, all other quantities of interest, such as ε_m/ν in the wall region, equation (2a), or the Fanning friction factor f

$$f = \frac{1}{2}\left(\int_0^1 (1-\eta)U^+(\eta)\,d\eta\right)^{-2} \quad (4)$$

can easily be predicted.

THE HEAT TRANSPORT

For the case studied the forced convection heat balance has the form

$$U(\eta)\frac{\partial\theta}{\partial\xi} - \frac{1}{1-\eta}\frac{\partial}{\partial\eta}((1-\eta)q) = 0, \quad \xi>0, \quad 0<\eta<1 \quad (5a)$$

where the diffusive heat flux q satisfies

$$q = \left(1+Pr\frac{\varepsilon_h}{\nu}\right)\frac{\partial\theta}{\partial\eta} \quad (5b)$$

and the eddy diffusivity ε_h/ν is

$$\frac{\varepsilon_h}{\nu} = \frac{1}{Pr_t}\frac{\varepsilon_m}{\nu}. \quad (5c)$$

The turbulent Prandtl number Pr_t is defined [3, 10] taking into account the correction for the roughness $\Delta\eta$, equation (2c), as

$$Pr_t = \begin{cases} 0.909B^+/26, & \eta=0 \\ 0.909\dfrac{1-\exp(-(\eta+\Delta\eta)R^+/26)}{1-\exp(-(\eta+\Delta\eta)R^+/B^+)}, & 0<\eta<\eta_w \\ 0.909, & \eta_w\leqslant\eta\leqslant 1 \end{cases} \quad (5d)$$

$$B^+ = Pr_f^{-0.5}\sum_{k=1}^{5} C_k(\log Pr_f)^{k-1}, \quad 0.02\leqslant Pr_f\leqslant 15 \quad (5e)$$

and $C_1 = 34.96$, $C_2 = 28.79$, $C_3 = 33.95$, $C_4 = 6.33$, $C_5 = -1.186$. Equations (5a) and (5b) yield the forced convection heat equation (Chap. 8 in ref. [13])

$$U(\eta)\frac{\partial\theta}{\partial\xi} = \frac{1}{1-\eta}\frac{\partial}{\partial\eta}\left((1-\eta)\left(1+Pr_f\frac{\varepsilon_h}{\nu}\right)\frac{\partial\theta}{\partial\eta}\right), \quad \xi>0, \quad 0<\eta<1 \quad (6a)$$

subject to boundary conditions

$$\theta(0,\eta) = \theta_0(\eta), \quad \xi=0, \quad 0\leqslant\eta\leqslant 1 \quad (6b)$$

$$\alpha\theta-\beta\left(1+Pr_f\frac{\varepsilon_h}{\nu}\right)\frac{\partial\theta}{\partial\eta} = \Phi(\xi), \quad \xi>0, \quad \eta=0 \quad (6c)$$

$$\frac{\partial\theta}{\partial\eta} = 0, \quad \xi>0, \quad \eta=1. \quad (6d)$$

In equations (6) ε_h/ν was defined by equations (5c)–(5e), θ is the appropriately scaled fluid temperature (Chap. 8 of ref. [13]) while α and β are constants used to simulate different boundary conditions at the tube wall, $\eta=0$; the cases of interest are: (a) $\alpha=1$, $\beta=0$, constant wall temperature and (b) $\alpha=0$, $\beta=1$, constant wall heat flux. For both cases it will be assumed in what follows that $\Phi(\xi)=1$ and $\theta_0(\eta)=0$.

It is shown in Chap. 8 of ref. [13] that for the above cases, far from the inlet of the tube ($\xi\to\infty$), the heat transfer coefficient for the fluid, $Nu_{\infty,f}$, can be calculated as (equations (8.41a) and (8.41d) of ref. [13])

$$Nu^T_{\infty,f} = 2\left(\frac{2}{\mu_1^2}-\frac{\beta}{\alpha}\right)^{-1}, \quad \alpha\neq 0 \quad (7a)$$

$$(Nu^H_{\infty,f})^{-1} = 2\int_0^1 \frac{\left(\int_{1-\eta}^1 (1-\eta)U^+(\eta)\,d\eta\right)^2}{(1-\eta)\left(1+Pr_f\dfrac{\varepsilon_h(\eta)}{\nu}\right)}d\eta, \quad \alpha=0 \quad (7b)$$

where μ_1^2 is the first eigenvalue of the linear Sturm–Liouville problem [5, 13, 14]

Table 1

N	$k_s/D \times 10^2$	k_s^+	Re_f	$f \times 10^2$ Ref. [1]	This work
1	1.634	23.0	19 500	1.037	1.030
2		77.3	63 000	1.109	1.060
3		223.9	179 900	1.122	1.085
4		811.0	646 000	1.140	1.027
5	0.833	34.4	65 000	0.843	0.835
6		57.9	107 900	0.867	0.841
7		216.8	398 000	0.887	0.863
8		335.7	615 000	0.889	0.857
9		500.0	916 000	0.897	0.843
10	0.397	15.0	66 000	0.632	0.625
11		41.9	178 000	0.682	0.669
12		150.5	628 000	0.705	0.685
13		291.3	970 500	0.700	0.687
14	0.198	6.9	66 100	0.532	0.524
15		10.0	97 100	0.552	0.510
16		20.3	192 000	0.537	0.532
17		69.5	612 000	0.581	0.556

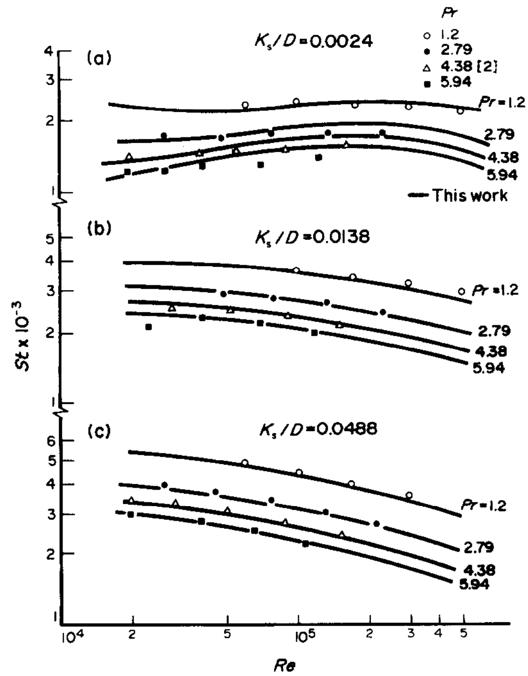

FIG. 1.

$$\frac{d}{d\eta}\left((1-\eta)\left(1+Pr_f\frac{\varepsilon_h}{\nu}\right)\frac{d\Psi}{d\eta}\right)+\mu^2(1-\eta)U(\eta)\Psi = 0,$$

$$0 < \eta < 1 \quad (8a)$$

$$\alpha\Psi - \beta\left(1+Pr_f\frac{\varepsilon_h}{\nu}\right)\frac{d\Psi}{d\eta} = 0, \quad \eta = 0 \quad (8b)$$

$$\frac{d\Psi}{d\eta} = 0, \quad \eta = 1 \quad (8c)$$

$$U(\eta) = U^+(\eta)/U_{av}^+ = u(r)/u_{av} \quad (8d)$$

$$U_{av}^+ = 2\int_0^1 (1-\eta)U^+(\eta)\,d\eta, \quad u_{av} = 2\int_0^1 ru(r)\,dr. \quad (8e)$$

To account for the roughness of the wall one has to represent $Nu_{\infty,f}$ in the form of a thermal resistance

Table 2

$k_s/D \times 10^2$	$Re_f \times 10^{-4}$	$Pr_f = 1.20$ Nu_*^H	Nu_*^T	$Pr_f = 2.79$ Nu_*^H	Nu_*^T	$Pr_f = 4.38$ Nu_*^H	Nu_*^T	$Pr_f = 5.94$ Nu_*^H	Nu_*^T
0	2.0	67.7	65.8	101.4	99.7	123.5	121.7	140.2	138.4
	5.0	138.6	135.6	215.1	212.1	266.0	263.0	304.8	301.8
	10.0	246.7	242.0	390.0	385.1	486.5	481.6	560.0	555.0
	20.0	424.5	417.2	684.1	676.5	860.1	852.6	994.6	986.9
	30.0	613.7	604.0	993.8	983.4	1253.6	1243.3	1451.8	1441.3
	50.0	927.7	914.0	1529.5	1515.2	1946.4	1931.9	2267.3	2253.0
0.24	2.0	56.9	55.5	92.7	91.1	119.8	118.2	142.6	140.9
	5.0	136.2	132.9	249.7	245.1	347.1	341.6	434.6	428.4
	10.0	289.3	280.8	552.9	539.7	774.8	758.5	969.9	951.0
	20.0	614.3	593.8	1154.8	1123.6	1598.8	1560.6	1982.4	1939.6
	30.0	839.7	870.3	1677.7	1633.5	2312.3	2258.6	2858.3	2797.7
	50.0	1428.2	1383.0	2639.5	2572.8	3620.4	3540.2	4461.3	4371.3
1.38	2.0	97.1	91.8	176.9	169.3	241.2	232.2	246.4	286.2
	5.0	232.9	220.4	415.1	397.7	559.3	539.1	681.7	659.5
	10.0	430.6	409.0	757.9	728.8	1015.6	982.1	1233.4	1196.8
	20.0	778.3	742.9	1360.0	1313.0	1816.1	1762.4	2200.9	2142.6
	30.0	1101.8	1053.9	1913.8	1850.9	2548.6	2477.1	3083.4	3006.0
	50.0	1690.3	1623.0	2916.0	2829.0	3871.2	3773.1	4674.9	4569.2
4.88	2.0	129.1	119.5	219.2	207.0	289.0	275.4	347.7	333.1
	5.0	278.5	260.7	469.4	447.2	616.9	592.3	740.5	714.3
	10.0	490.6	462.3	822.7	787.9	1078.8	1040.3	1293.2	1252.3
	20.0	846.2	802.7	1413.8	1360.8	1850.9	1792.7	2216.8	2154.9
	30.0	1166.9	1110.1	1941.0	1872.4	2536.3	2461.3	3034.3	2954.9
	50.0	1728.8	1651.6	2865.7	2773.2	3739.1	3638.3	4469.4	4362.8

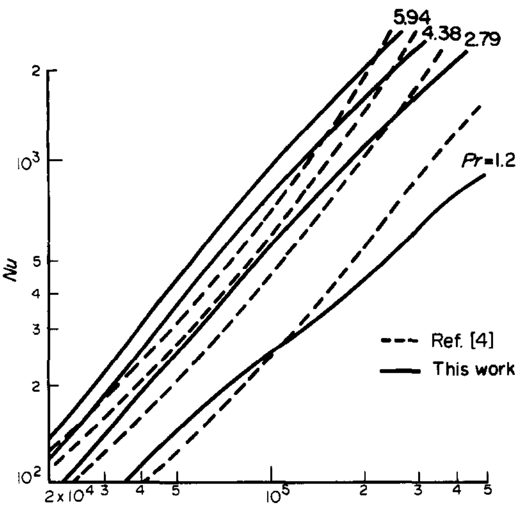

FIG. 2.

and add to it the thermal resistance of the fluid layer enclosing the roughness elements (even in the absence of a viscous sublayer). Such a resistance has been correlated [2] in the form of a Stanton number

$$(St_w)^{-1} = 5.19(2/f)^{0.5}(k_s^+)^{0.2}\,Pr_f^{0.44}. \quad (9)$$

Having in mind the standard definition of the Stanton number, equations (7) and (9) finally yield

$$Nu_{\infty,f} = Re_f\,Pr_f((St_w)^{-1}+(St_{\infty,f})^{-1})^{-1}. \quad (10)$$

Equations (3) and (8) cannot be solved analytically and have to be integrated by numerical methods. Equations (3) were processed with a standard variable step Runge–Kutta method (for a fixed iterate value of R^+ while η_w was currently calculated from the continuity condition for ε_m/v) restricting both the increments of U^+ and η. The calculation of $Nu^T_{\infty,f}$ through the solution of the eigenvalue problem, equations (8) (for sand-roughened tubes such an approach was used earlier in ref. [5]), was realized via the 'sign-count' method [14] using a fixed resolution of the interval [0, 1] generated in the course of solution of equations (3). Other details concerning the numerical integration of equations (3) and (8) can be found in refs. [10, 11]. The results reported in the next section were calculated with a BASIC code run on an IBM/AT (relative precision 5.96×10^{-8}).

RESULTS AND DISCUSSION

The numerical results for the Fanning friction factor f, equation (4), are presented in Table 1 together with the experimental data [1]. As seen from Table 1 the percentage difference is less than 5% excluding positions 4 and 9. In contrast, in ref. [8] where the same correction $\Delta\eta$, equation (2c), was used in conjunction with another model for the velocity distribution, equations (3), an average difference of 15% was observed.

The calculations for Nu^H_∞ and Nu^T_∞ are summarized in Table 2. The values for Nu^H_∞ (transformed as Stanton numbers) are compared with the experimental data [2] on Figs. 1(a)–(c) and to the calculations (Fig. 7.6 of ref. [4]) on Fig. 2. As seen from Fig. 1 there is a fairly good agreement between our calculations and the experimental plots [2] although a quantitative estimate is not available. A relatively higher divergence is observed only for $k_s/D = 0.0024$ and $Pr_f = 4.38$ and 5.94. From Fig. 2 it is seen that the model [4] (plots for other values of k_s/D are not published there) for $Re_f > 200\,000$ yields considerably higher values for the heat transfer coefficient than the model described here, which favours the latter.

The calculated values for Nu^H_∞ and Nu^T_∞, Table 2 (scaled as Stanton numbers), satisfy a condition published (without comments) elsewhere (Chap. 7 of ref. [15]), namely, that $Nu^H_\infty > Nu^T_\infty$ and the (relative) difference between these two quantities decreases as Pr_f increases.

It can be concluded that the model of stabilized momentum and heat transfer in turbulent incompressible sand-roughened wall tube flow discussed yields numerical results which agree better with standard experimental data and it can be considered as improving earlier calculations [3, 5, 8].

REFERENCES

1. J. Nikuradse, Stromungsgesetze in rauhen Rohren, *VDI Vorschungshefte* No. 4, 1–22 (1933).
2. D. F. Dipprey and R. H. Saberski, Heat and momentum transfer in smooth and rough tubes at various Prandtl numbers, *Int. J. Heat Mass Transfer* **6**, 329–353 (1963).
3. T. Cebeci and A. M. O. Smith, *Analysis of Turbulent Boundary Layers*. Academic Press, New York (1974).
4. T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*. Springer, New York (1984).
5. P. Darmono and Yu. P. Kouznetsov, Hydrodynamics, heat and mass transfer at moderate Prandtl numbers in turbulent pipe flow with smooth and rough walls, *Theor. Fund. Chem. Tech.* **XIV**(3), 333–341 (1980) (in Russian).
6. P. M. Lingran, W. M. Kays and R. J. Moffat, A heat transfer prediction method for turbulent boundary layers developing over rough surfaces with transpiration, *Int. J. Heat Mass Transfer* **24**, 774–778 (1981).
7. A. P. Hatton and R. J. Walklate, A mixing length method for predicting heat transfer in rough pipes, *Int. J. Heat Mass Transfer* **19**, 1425–1431 (1976).
8. S. K. Momoh, An eddy-viscosity approach to the calculation of velocity and heat-transfer properties of flow in rough pipes, *Lett. Heat Mass Transfer* **6**, 125–130 (1979).
9. E. R. van Driest, On turbulent flow near a wall, *J. Aeronaut. Sci.* **23**, 1007–1014 (1956).
10. M. D. Mikhailov, N. L. Vulchanov and V. D. Zimparov, Computation of stabilized turbulent fluid flow and heat transfer in circular smooth pipes for moderate Prandtl numbers. Part 1. Fluid flow and eddy diffusivity, *Theor. Appl. Mech.* **XVI**(2), 55–59 (1985).
11. M. D. Mikhailov, N. L. Vulchanov and V. D. Zimparov, Computation of stabilized turbulent fluid flow and heat

transfer in circular smooth pipes for moderate Prandtl numbers. Part 2. The limiting heat transfer coefficients, *Theor. Appl. Mech.* **XVI**(4), 49–54 (1985).
12. M. de Pinho and R. W. Fahien, Correlation for the eddy diffusivities for pipe flow, *A.I.Ch.E. Jl* **27**, 170–172 (1981).
13. M. D. Mikhailov and M. N. Ozisik, *Unified Analysis and Solutions of Heat and Mass Diffusion*. Wiley, New York (1984).
14. M. D. Mikhailov and N. L. Vulchanov, A computational procedure for Sturm–Liouville problems, *J. Comp. Phys.* **50**, 323–332 (1983).
15. W. M. Rosenow and J. P. Hartnett, *A Handbook of Heat Transfer*. McGraw-Hill, New York (1973).

FROTTEMENT ET TRANSFERT THERMIQUE TURBULENTS DANS DES TUBES CIRCULAIRES A RUGOSITE INTERNE PAR DU SABLE ET POUR DES NOMBRES DE PRANDTL MODERES

Résumé—On propose un modèle à longueur de mélange et des résultats numériques pour le coefficient de frottement et le coefficient de convection—le nombre de Nusselt—pour un écoulement forcé turbulent, incompressible, monophasique, hydrodynamiquement et thermiquement stabilisé, dans un tube cylindrique à paroi rugueuse par du sable, pour des nombres de Prandtl modérés. On simule les conditions aux limites de flux thermique pariétal constant et de température pariétale uniforme. Une comparaison des résultats et d'autres données expérimentales et numériques déjà publiées montre que l'accord est meilleur qu'avec d'autres calculs antérieurs.

REIBUNG UND WÄRMEÜBERGANG BEI AUSGEBILDETER TURBULENTER STRÖMUNG IN ZYLINDRISCHEN ROHREN MIT SANDARTIGER RAUHIGKEIT BEI MITTLEREN PRANDTL-ZAHLEN

Zusammenfassung—Es wird ein Modell vom Typ des Mischungswegansatzes vorgeschlagen. Reibungsbeiwert und Wärmeübergangskoeffizient (Nusselt-Zahl) werden numerisch für eine hydrodynamisch und thermisch ausgebildete inkompressible turbulente erzwungene Konvektionsströmung in einem runden Rohr mit sandartiger Rauhigkeit bei mittleren Prandtl-Zahlen berechnet und dargestellt. Als Randbedingungen werden sowohl konstanter Wärmestrom als auch konstante Wandtemperatur simuliert. Der Vergleich dieser Ergebnisse mit anderen verfügbaren experimentellen und numerischen Daten zeigt eine bessere Übereinstimmung als früher veröffentlichte Berechnungen.

СТАБИЛИЗИРОВАННОЕ ТУРБУЛЕНТНОЕ ГИДРОДИНАМИЧЕСКОЕ ТРЕНИЕ И ТЕПЛОПЕРЕНОС В КРУГЛЫХ ТРУБАХ С ПЕСЧАНОЙ ШЕРОХОВАТОСТЬЮ ВНУТРЕННИХ СТЕНОК ПРИ УМЕРЕННЫХ ЧИСЛАХ ПРАНДТЛЯ

Аннотация—Предложена модель длины смешения, а также представлены численные данные для коэффициента трения фаннинга и коэффициента теплопереноса (число Нуссельта) для гидродинамически и термически стабилизированного несжимаемого турбулентного вынужденного конвективного потока в круглой трубе се песчаной шероховатостью внутренних стенок при умеренных числах Прандтля. Воспроизводятся граничные условия постоянного теплового потока и постоянной температуры. Сравнение представленных результатов с другими экспериментальными и численными данными обнаруживает лучшее соответствие, чем в ранее опубликованных расчетах.